5/10/17 "Eighty percent of success is showing up." -Woody Allen

HW: "Sinusoidal Modeling" homework section
Test 2 on Thursday 5/18

AIM: How do we model real world situations using sinusoidal graphs?

Warm Up:

cosecant

Find the exact value of csc 150°

reciprocal of sin

$\sin 150 = \frac{1}{2}$

$\csc 150 = \frac{2}{1}$

SINUSOIDAL MODEL COEFFICIENTS

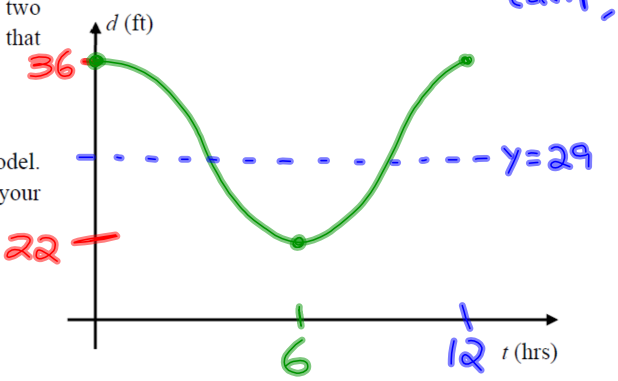
For $y = A\sin(Bx)+C$ and $y = A\cos(Bx)+C$

$\lvert A\rvert$	the **amplitude** or distance the sinusoidal model rises and falls above its midline
C	the **midline** or average y-value of the sinusoidal model
B	the **frequency** of the sinusoidal model – related to the **period**, P, by the equation $BP = 2\pi$
P	the **period** of the sinusoidal model – the minimum distance along the x-axis for the cycle to repeat

Exercise #1: The tides in a particular bay can be modeled with an equation of the form $d = A\cos(Bt)+C$, where t represents the number of hours since high-tide and d represents the depth of water in the bay. The maximum depth of water is 36 feet, the minimum depth is 22 feet and high-tide is hit every 12 hours.

high tide - when the water is highest (deeper)

(a) On the axes, sketch a graph of this scenario for two full periods. Label the points on this curve that represent high and low tide.



(b) Determine the values of A, B, and C in the model. Verify your answers and sketch are correct on your calculator.

$A = \frac{36-22}{2} = \boxed{7}$

$C = \frac{36+22}{2} = \boxed{29}$

$B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{12} = \frac{\pi}{6}$

(c) Tanker boats cannot be in the bay when the depth of water is less or equal to 25 feet. Set up an inequality and solve it graphically to determine all points in time, t, on the interval $0 \le t \le 24$ when tankers cannot be in the bay. Round all times to the nearest *tenth* of an hour.

$y = 7\cos\left(\frac{\pi}{6}x\right) + 29$

$4.2 \rightarrow 7.8$

$16.2 \rightarrow 19.8$

cannot go in @

$(4.2, 7.8)$

$(16.2, 19.8)$

***Exercise* #2:** The height of a yo-yo above the ground can be well modeled using the equation $h = 1.75\cos(\pi t) + 2.25$ where h represents the height of the yo-yo in feet above the ground and t represents time in seconds since the yo-yo was first dropped from its maximum height.

(a) Determine the maximum and minimum heights that the yo-yo reaches above the ground. Show the calculations that lead to your answers.

max $= 1.75\cos(\pi(0)) + 2.25$
$= 4$

Alt:
Max $= C + A = 4$ ft
Min $= C - A = .5$ ft

(b) How much time does it take for the yo-yo to return to the maximum height for the first time?

(Period) Freq $= \pi$

Period $= \dfrac{2\pi}{\text{Freq}}$

Period $= \dfrac{2\pi}{\pi} = 2$

2 seconds

***Exercise* #3:** A Ferris wheel is constructed such that a person gets on the wheel at its lowest point, five feet above the ground, and reaches its highest point at 130 feet above the ground. The amount of time it takes to complete one full rotation is equal to 8 minutes. A person's vertical position, y, can be modeled as a function of time in minutes since they boarded, t, by the equation $y = A\cos(Bt) + C$. Sketch a graph of a person's vertical position for one cycle and then determine the values of A, B, and C. Show the work needed to arrive at your answers.

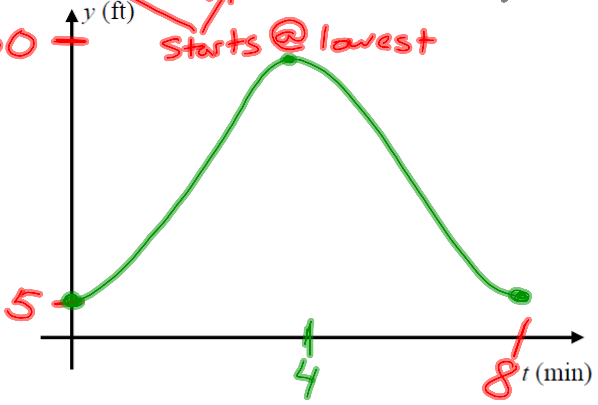
$A = -62.5$ $y = -62.5\cos\left(\frac{\pi}{4}t\right) + 67.5$

Starts @ lowest

$|A| = \dfrac{130-5}{2} = 62.5$

$C = \dfrac{130+5}{2} = 67.5$

$B = \dfrac{2\pi}{\text{Period}} = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$



***Exercise* #4:** The possible hours of daylight in a given day is a function of the day of the year. In Poughkeepsie, New York, the minimum hours of daylight (occurring on the Winter solstice) is equal to 9 hours and the maximum hours of daylight (occurring on the Summer solstice) is equal to 15 hours. If the hours of daylight can be modeled using a sinusoidal equation, what is the equation's amplitude?

(1) 6
(2) 12
(3) 3
(4) 4

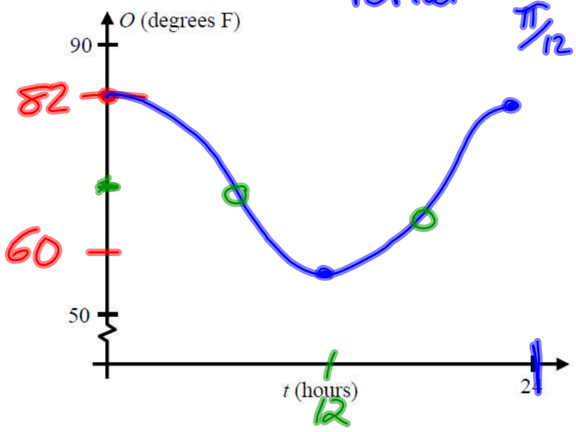
$|A| = \dfrac{15-9}{2} = \dfrac{6}{2} = 3$

3. On a standard summer day in upstate New York, the temperature outside can be modeled using the sinusoidal equation $O(t) = 11\cos\left(\frac{\pi}{12}t\right) + 71$, where t represents the number of hours since the peak temperature for the day.

$71 + 11 = 82$ $71 - 11 = 60$

(a) Sketch a graph of this function on the axes below for one day.

Period $= \frac{2\pi}{\pi/12}$



(b) For $0 \le t \le 24$, graphically determine all points in time when the outside temperature is equal to 75 degrees. Round your answers to the nearest tenth of an hour.

$t = 4.6$
and
$t = 19.4$

4. The percentage of the moon's surface that is visible to a person standing on the Earth varies with the time since the moon was full. The moon passes through a full cycle in 28 days, from full moon to full moon. The maximum percentage of the moon's surface that is visible is 50%. Determine an equation, in the form $P = A\cos(Bt) + C$ for the percentage of the surface that is visible, P, as a function of the number of days, t, since the moon was full. Show the work that leads to the values of A, B, and C.

5. Evie is on a swing thinking about trigonometry (no seriously!). She realizes that her height above the ground is a periodic function of time that can be modeled using $h = 3\cos\left(\frac{\pi}{2}t\right) + 5$, where t represents time in seconds. Which of the following is the range of Evie's heights?

(1) $2 \le h \le 8$ (3) $3 \le h \le 5$

(2) $4 \le h \le 8$ (4) $2 \le h \le 5$ ________

1. A ball is attached to a spring, which is stretched and then let go. The height of the ball is given by the sinusoidal equation $y = -3.5\cos\left(\frac{4\pi}{5}t\right) + 5$, where y is the height above the ground in feet and t is the number of seconds since the ball was released.

(a) At what height was the ball released at? Show the calculation that leads to your answer.

$$\boxed{y_{min} = 5 - 3.5 = 1.5 \text{ ft}}$$

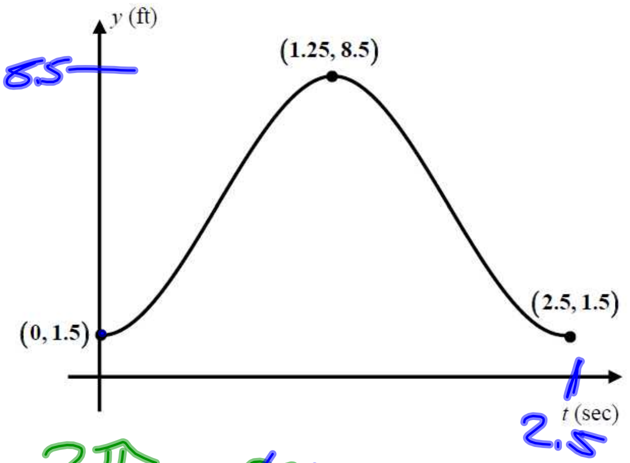
(b) What is the maximum height the ball reaches?

$$\boxed{y_{max} = 5 + 3.5 = 8.5 \text{ ft}}$$

(c) How many seconds does it take the ball to return to its original position?

$$\boxed{\frac{4\pi}{5}P = 2\pi \Rightarrow P = 2\pi \cdot \frac{5}{4\pi} = \cancel{2\pi} \cdot \frac{5}{2 \cdot \cancel{2\pi}} = \frac{5}{2} = 2.5 \text{ sec}}$$

(d) Draw a rough sketch of one complete period of this curve below. Label maximum and minimum points.



$$\frac{2\pi}{\frac{4\pi}{5}} \qquad \cancel{2\pi} \cdot \frac{5}{\cancel{4\pi}_{2}} = \frac{5}{2} = 2.5$$

2. An athlete was having her blood pressure monitored during a workout. Doctors found that her maximum blood pressure, known as systolic, was 110 and her minimum blood pressure, known as diastolic, was 70. If each heartbeat cycle takes 0.75 seconds, then determine a sinusoidal model, in the form $y = A\sin(Bt) + C$, for her blood pressure as a function of time t in seconds. Show the calculations that lead to your answer.

$$C = \frac{70 + 110}{2} = \frac{180}{2} = 90$$

$$A = \frac{110 - 70}{2} = \frac{40}{2} = 20$$

$$0.75B = 2\pi \Rightarrow B = \frac{2\pi}{0.75} = \frac{2}{0.75}\pi = \frac{8}{3}\pi$$

$$\Rightarrow \boxed{y = 20\sin\left(\frac{8\pi}{3}t\right) + 90}$$

$$\frac{2\pi}{.75} = \frac{2}{.75}\pi = \frac{8}{3}\pi \text{ or } \frac{8\pi}{3}$$